ISSN No.: 2455-6130
Volume 2, Issue 5, 2017, pp. 27-33

website: www.ijoetr.com

International Journal of Emerging Trends in Research

Bulk Service Queueing Analysis with Removable Server and Working Vacation

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Abstract

Arrival pattern of the customer is one by one and there is single service point. Service time distribution is exponential and queue discipline is first come first serve basis. We will find out expected queue length when server will empty and will be in working state on removable server.

Keywords: Bulk; Server; Probability; Bulk

1. Introduction

The In this Paper, the arrivals arrive one by one and service by a single server. If the Queue is empty then server goes on vacation, when Queue size is less than 'a' then server serve the customer with rate μ_1 and if the Queue size is greater than 'a' then server serve the customer with a batch size 'd' of rate μ_2.

The following assumption describe the system

1) Arrivals arrive according to poisson law with parameter λ
2) The service time distribution is exponentially with rate μ_1 and μ_2
3) Server serve the customer of batch sized if server is busy
4) Server serve the customer single if batch size is less than 'a'
5) The Queue discipline is first come first serve
6) The various stochastic process in the system are statistically independent
7) The server is removed from its service as soon as the Queue is empty.

Notations

$P_{00}(t)$: Probability that there are no arrival and server is on vacation.

$P_{i1}(t)$: Probability that there are exactly 'i' arrival and server is an working vocation

$P_{i2}(t)$: Probability that there are exactly 'i' arrivals and server is busy

$X(t) = 0$ $y(t) = 0$ removal state

$x(t) < a$ $y(t) = 1$ working vacation

$xt \geq a$ $y(t) = 2$ busy.

The difference-differential equations governing the mode are

$$P'_{00}(t) = -\lambda P_{00}(t) + \mu_1 P_{0,1}(t) + \mu_2 P_{02}(t) \quad \ldots(1)$$

$$P'_{01}(t) = -(\lambda + \mu_1) P_{0,1}(t) + \lambda P_{00}(t) + \mu_1 P_{11}(t) + \mu_2 d P_{12} \quad \ldots(2)$$

$$P'_{n1}(t) = -(\lambda + \mu_1) P_{n,1}(t) + \lambda P_{n-1}(t) + \mu_1 P_{n+1,1}(t) \quad n \leq a \quad \ldots(3)$$

$$P'_{0,2}(t) = (\lambda + d \mu_2) P_{0,2}(t) + \lambda P_{a+1,1}(t) + d\mu_2 P_{n,2}(t) \quad \ldots(4)$$

$$P'_{n,2}(t) = (\lambda + d \mu_2) P_{n,2}(t) + \lambda P_{n,2}(t) + d \mu_2 P_{n+a,2}(t) \quad n > a \quad \ldots(5)$$

Taking Laplace Transformation of Equation (1) – (5) we get

$$S\overline{P}_{00}(s) - P_{00}(s) = -\lambda \overline{P}_{00}(s) + \mu_1 \overline{P}_{01}(s) + \mu_2 \overline{P}_{02}(s)$$

$$\overline{P}_{00}(s) = \frac{1}{S+\lambda}(1 - \mu_1 \overline{P}_{01}(s) + \mu_2 \overline{P}_{02}(s))$$

$$S\overline{P}_{01}(s) - P_{01}(s) = -(\lambda + \mu_1)\overline{P}_{01}(s) + \lambda \overline{P}_{00}(s) + \mu_1 \overline{P}_{11}(s) + \mu_2 d\overline{P}_{12}(s)$$

$$\overline{P}_{01}(s) = \frac{1}{(S+\lambda+\mu_1)}[\lambda \overline{P}_{00}(s) + \mu_1 \overline{P}_{11}(s) + \mu_2 d\overline{P}_{12}(s)]$$

$$\overline{P}_{n1}(s) = \frac{1}{S+\lambda+\mu_1}[\lambda \overline{P}_{n-1,1}(s) + \mu_1 \overline{P}_{n+1,1}(s) + \mu_2 d\overline{P}_{n+1,2}(s)]$$

$$S\overline{P}_{02}(s) - P_{0,2}(0) = -(\lambda + d\mu_2)\overline{P}_{02}(s) + \lambda \overline{P}_{a+1,1}(s) + d\mu_2 \overline{P}_{n,2}(s)$$

$$\overline{P}_{02}(s) = \frac{1}{S+\lambda+d\mu_2}[\lambda \overline{P}_{a+1,1}(s) + d\mu_2 \overline{P}_{n,2}(s)]$$

$$(S+\lambda+d\mu_2)\overline{P}_{n,2}(s) = \lambda \overline{P}_{n,2}(s) + d\mu_2 \overline{P}_{n+a,2}(s)$$

$$h(z) = d \mu_2 z^{a+1} - (s + \lambda + d\mu_2) z + \lambda h(z) = 0$$

$$d\mu_2 z^{a+1} - (S + \lambda + d\mu_2) z + \lambda = 0$$

Suppose that

$f(z) = -(S + \lambda + d\,\mu_2)^2$

$g(z) = d\mu_2\, z^{a+1} + \lambda$

Consider $|z| = 1 - \delta$

$|g(z)| < |f(z)|$

then f(2) and {f(z) + g(z)} will have same number of zero as inside z = 1 – δ roots of h(z) = 0 is real and unique iff

$$P = \frac{\lambda}{\mu_1} < 1$$

$$Z = \frac{(S+\lambda+d\mu_2) \pm \sqrt{(S+\lambda+d\mu_2)^2 - 4\mu_2\lambda d}}{2d\mu_2}$$

$$\alpha = (S + \lambda + d\,\mu_2) + \sqrt{(S+\lambda+d\mu_2)^2 - 4\mu_2 d\lambda}$$

$$\beta = (S + \lambda + d\,\mu_2) - \sqrt{(S+\lambda+d\mu_2)^2 - 4\mu_2 d\lambda}$$

Let α is the unique positive real part

$$\overline{P}_{n1}(s) = \overline{P}_{01}(s)\alpha - \overline{P}_{02}(s)\ d\mu_2\, R^n$$

$$\overline{P}_{02}(s) = \frac{\overline{P}_{01}(s)\alpha\left(\frac{\lambda}{S+\lambda+\mu_1}\right)\left(\frac{\lambda}{(S+\lambda+d\mu_2}\right)}{\left[1 + d\mu_2\left(\frac{\lambda}{S+\lambda+\mu_1}\right)\left(\frac{\lambda}{S+\lambda+\mu_2}\right) < R^n\right]}$$

$$\overline{P}_{00}(s) = \frac{\mu_1}{S+\lambda}\,\overline{P}_{01}(s) + \frac{d\mu_2}{S+\lambda}\,\overline{P}_{01}(s)\ D_1 + \frac{1}{S+\lambda}\ + d\,\mu_2\,\lambda^2\,\alpha\,R^n$$

$$D_1 = \frac{\lambda^2\alpha}{(S+\lambda+\mu_1)(S+\lambda+d\mu_2)(d\mu_2\lambda^2\alpha R^n)}$$

$$\overline{P}_{00}(s) = \frac{1}{(S+\lambda)}[(\mu_1 + d\mu_2 D_1)\overline{P}_{01}(s) + 1]$$

$$\overline{P}_{n1}(s) = [\alpha - d\,\mu_2\,D_1\,R^{n+1}]\ \overline{P}_{01}(s)$$

$$\overline{P}_{n,2}(s) = \frac{\alpha\lambda^2 R^n}{(S+\lambda+\mu_1)(S+\lambda+d\mu_2) + d\mu_2\lambda^2\alpha R^n}\,\overline{P}_{01}(s)$$

According to normalizing condition

$$\sum_{i=0}^{n}\overline{P}_{i0}(s)+\overline{P}_{i1}(s)+\overline{P}_{i2}(s)=\frac{1}{S}$$

steady state probabilities

$$P_{in}=\lim_{s\to 0} S\,\overline{P}_{1n}(s)$$

$$P_{00}=\left[\frac{\mu_1}{\lambda}+\frac{d\mu_2}{\lambda}k\right]P_{01}$$

$$K=\frac{\lambda^2\alpha}{(\lambda+\mu_1)(\lambda+d\mu_2)+d\mu_2\lambda^2\alpha R^n}$$

$$P_{n1}=\left[\left(\frac{\lambda}{\mu_1}\right)^n-k\mu^2R^n\right]P_{01}$$

$$P_{n2}=P_{01}\left[\frac{\lambda^2\alpha R^n}{(\lambda+\mu_1)(\lambda+\mu_2 d)+d\mu_2 a\lambda^2\alpha R^n}\right]$$

Expected Queue Length if server is removed

$$L_{qR}=\sum_{i=0}^{n}n\,P_{00}=0$$

Hence it verify that server is on removed state of these is no customer is the queue.

Expected Queue length if server is on working vacation

$$L_{q_{wv}}=\sum_{n=1}^{a}n\;P_{n1}$$

$$L_{q_{wv}}=P_{01}a\left\{\sum_{n=1}^{a-1}n\left(\frac{\lambda}{\mu_1}\right)^n-d\mu_2R^n\frac{\lambda^2\alpha}{(\lambda+\mu_1)(\lambda+d\mu_2)+d\mu_2\lambda^2\alpha R^n}\right\}$$

Expected Queue length at busy period

$$L_{qB}=\sum_{n=a}^{\infty}n\;P_{n2}$$

$$L_{qB}=P_{01}\sum_{n=a}^{\infty}n\frac{\lambda^2\alpha R^n}{(\lambda+\mu_1)(\lambda+d\mu_2)+d\mu_2\lambda^2\alpha R^n}$$

Table 1. Value of working vacation queue length or busy period queue length

P01	D	λ	μ1	μ2	a	Lqwv
0.06	0.2	0.1	0.2	0.2	0.125	0.005584592
0.06	0.2	0.1	0.2	0.3	0.125	0.005099943
0.06	0.2	0.1	0.2	0.4	0.125	0.004430666
0.06	0.2	0.1	0.2	0.5	0.125	0.003576759

2. Numerical Results

Table 1 and 2 shows the value of working vacation queue length or busy period queue length at different arrival rates and service rates. Figure 1 shows the behavior of working vacation queue length. It is clear from the graph that if service rate is increases then working vacation queue length is decreases at different batch service. Figure 2 shows the behavior of working vacation queue length. It is clear from the graph that if arrival rate λ is increases then queue length is also increases.

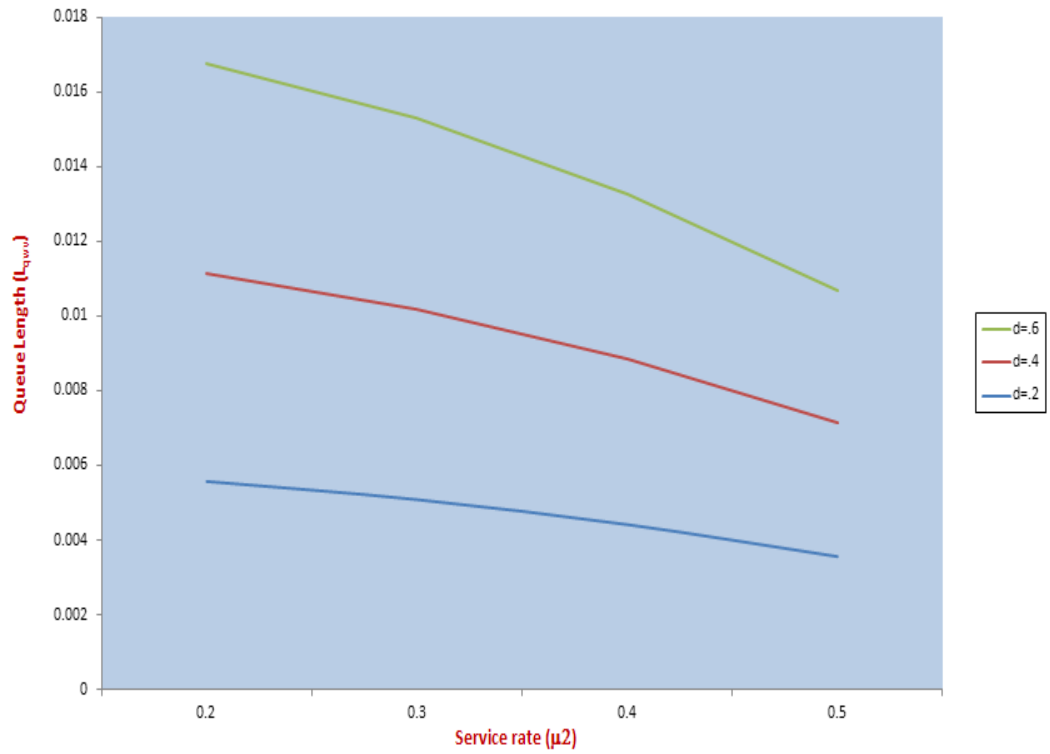

Figure 1. Service rate vs Queue Length

Table 2. Value of working vacation queue length or busy period queue length

P_{01}	D	λ	μ_1	μ_2	a	L_{qwv}
0.06	0.2	0.1	0.4	0.6	0.125	0.00985814
0.06	0.2	0.2	0.4	0.6	0.125	0.016475574
0.06	0.2	0.3	0.4	0.6	0.125	0.020746317

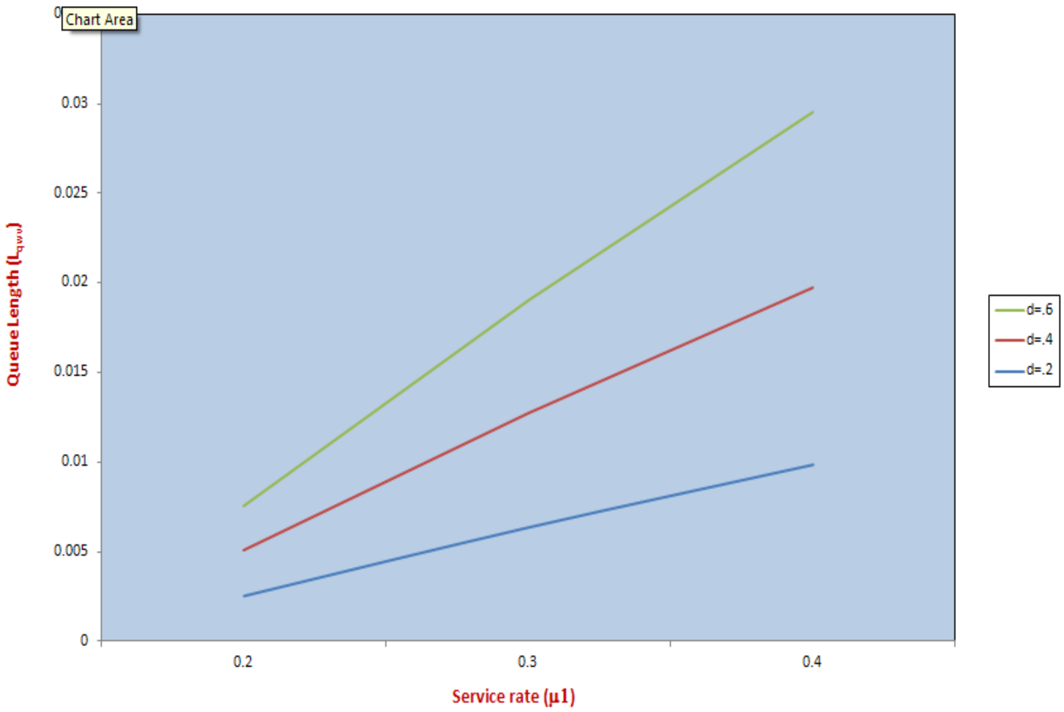

Figure 2. Service rate vs Queue Length

3. Conclusions

Conclusion It is clear from the Figure 1 that if service rate is increases then working vacation queue length is decreases at different batch service It is clear from the Figure 2 that as batch size increases queue length in working vacation is also increases.

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